EXPERIMENTAL TEST OF COUPLED WAVE MODEL OF LARGE COILS

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Abstract: Recent data from Time Domain Pulse Reflectometry experiments on a three turn coil in the form of a race track corroborate the theory of coupled wave model for large coils. The data demonstrate effects of turn-to-turn coupling, pulse travel time, reduced damping at liquid Nitrogen temperature, and the relation between short-term and long-term transient behavior. The use of large superconductor coils as energy storage elements inherently poses the problem of dealing with fast electromagnetic transients in the vicinity of superconductor boundaries. Treatment of such coils as the lumped inductors of circuit theory is grossly inaccurate when events to be reckoned are on nanosecond scale. Rigorous treatment from the standpoint of electrodynamic field theory has resulted in the Coupled Wave Model which is based on the theory of multiple conductor transmission structures. The experimental data illustrate the merit of the model for analysis of transients on large coils and motivate further refinement and extension.

Introduction

One of the important applications of superconductivity is found in large dc magnets, which might be utilized as the primary source of very strong steady magnetic fields or as an energy storage element in pulse power generators[1]. The large scale of such magnets in all respects --physical size, current, and magnetic energy --introduce new problems both in electrodynamic and thermal behavior which normally do not pose difficulties in small scale applications, though they might be present to a lesser degree. One such problem, which the designer of any large magnet must face, is electromagnetic transients and the likelihood of high voltage breakdown together with the damage that results from them.

The occurrence of high voltage breakdown between layers of windings of large coils is not new since it has been one of the lingering problems in the design of large power transformers. That breakdown should occur in alternating current transformers is not so surprising from a theoretical standpoint. However, at first glance, it would seem very unusual that it should also occur in direct current superconducting magnets. Yet, evidence of high voltage breakdown has been observed in such magnets following emergency shut down when the stored magnetic energy is dumped into shunt resistors in accordance with the conventional way of limiting voltages across inductors. In the conventional viewpoint, a coil of wire is treated as the lumped inductor of circuit theory. According to this theory, during the transient current decay period, the voltage across the terminals and anywhere within the coil should never exceed the voltage across the limiting shunt resistor. Thus, it is clear that the conventional circuit representation of a coil in not adequate to account for high voltages. A better understanding of electromagnetic transients is needed.

The inadequacy of the conventional approach lies in the fact that any circuit representation of a physical system, by nature having no provision for spatial distribution, treats all events simultaneously. In reality, however, electromagnetic events do not occur simultaneously at any pair of points separated by a distance. The bearing of this fact on the transient behavior of large coils is appreciated when one considers that an electromagnetic disturbance travels one foot in one nanosecond. Consequently, circuit theory is grossly inappropriate to attack transients on large scale. It soon became apparent that the key to understanding such transients lies in field theoretic modelling. In an earlier paper, Gabriel and Burkhart[2] reported observation of multiplicity of resonances in frequency as well as evidence of multiple wave reflections in time. A very crude analysis based on treating the coil as a single wire transmission line yielded surprisingly good qualitative agreement. Based on this success, attention was directed to laying the foundations for the more realistic Coupled Wave Model the details of which are given in an earlier document[3]. In this paper, a brief account is given of further development of the model, and recent data are presented as a preliminary step toward corroboration and further evolution.

Theoretical Foundation

Outline of Model

The objective has been to evolve a model which is both rigorously founded on field theory and broadly compatible with experimental evidence. The most suggestive evidence is the observed travel time of a disturbance around a single turn, because it indicates that the transient field behaves very nearly as a Tranverse ElectroMagnetic (TEM) wave. Motivated by this fact, the model is founded on the theory of multiple conductor transmission lines which support TEM waves, even though the geometry of any coil is not exactly compatible with the conditions for TEM waves in the strict sense. The gist of the model is conveyed by a wave coupling matrix, conveniently called an inductance or capacitance matrix, and the crucial interconnection matrix containing the naturally periodic boundary conditions demanded by a coil structure. With ζ measuring the coordinate distance along any single turn of length 2L, the electric and magnetic fields on the surface of the wire of the i^{th} turn are characterized by amplitude functions $v_i(\zeta,t)$ and $i_i(\zeta,t)$, conveniently called voltage and current. A coil of M turns is then described by the M-dimensional column vectors $\underline{v}(\zeta,t)$ and $\underline{i}(\zeta,t)$ which satisfy the standard telegraphist equations

$$\frac{\partial}{\partial\zeta}\underline{v}(\zeta,t) = -\mu\overline{\gamma}^{-1}\frac{\partial}{\partial t}\underline{i}(\zeta,t) \qquad (1)$$

$$\frac{\partial}{\partial\zeta}\underline{i}(\zeta,t) = -\,\varepsilon\,\overline{\gamma}\frac{\partial}{\partial t}\underline{v}(\zeta,t) \qquad (2)$$

where $\overline{\gamma}$ is the symmetric coupling matrix. These equations result from appropriate normalization of the field equations for the vector potential $\underline{A}(\zeta,t)$ and the scalar Coulomb potential $\phi(\zeta,t)$, which are actually components of the Lorentz invariant potential in 4-space.

After Laplace transformation, and denoting the transforms as $\underline{V}(\zeta,s)$ and $\underline{I}(\zeta,s)$, one finds the general solution to be, as expected, a set of waves travelling in unison, viz.

Report Documentation Page			Form Approved OMB No. 0704-0188

1. REPORT DATE	2. REPORT TYPE	3. DATES COVERED
JUN 1985	**N/A**	-

4. TITLE AND SUBTITLE	
Experimental Test Of Coupled Wave Model Of Large Coils	5a. CONTRACT NUMBER
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Department of Electrical Engineering University of Notre Dame, Notre Dame, IN 46556**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR’S ACRONYM(S)
	11. SPONSOR/MONITOR’S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES

See also ADM002371. 2013 IEEE Pulsed Power Conference, Digest of Technical Papers 1976-2013, and Abstracts of the 2013 IEEE International Conference on Plasma Science. Held in San Francisco, CA on 16-21 June 2013. U.S. Government or Federal Purpose Rights License.

14. ABSTRACT

Recent data from Time Domain Pulse Reflectometry experiments on a three turn coil in the form of a race track corroborate the theory of coupled wave model for large coils. The data demonstrate effects of turn-to-turn coupling, pulse travel time, reduced damping at liquid Nitrogen temperature, and the relation between short-term and long-term transient behavior. The use of large superconductor coils as energy storage elements inherently poses the problem of dealing with fast electromagnetic transients in the vicinity of superconductor boundaries. Treatment of such coils as the lumped inductors of circuit theory is grossly inaccurate when events to be reckoned are on nanosecond scale. Rigorous treatment from the standpoint of electrodynamic field theory has resulted in the Coupled Wave Model which is based on the theory of multiple conductor transmission structures. The experimental data illustrate the merit of the model for analysis of transients on large coils and motivate further refinement and extension.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a REPORT	b ABSTRACT	c THIS PAGE			
unclassified	**unclassified**	**unclassified**	**SAR**	**5**	

$$\underline{V}(\zeta,s) = \underline{A}e^{-s\zeta/c} + \underline{B}\ e^{s\zeta/c} \quad (3)$$

$$\underline{I}(\zeta,s) = \overline{Y}(\underline{A}e^{-s\zeta/c} - \underline{B}\ e^{\zeta s/c}), \quad (4)$$

where $\underline{A}$ and $\underline{B}$ are column vectors to be determined by boundary conditions, and $\overline{Y} = \sqrt{\varepsilon/\mu}\ \overline{\gamma}$ is a characteristic wave admittance which is proportional to the geometric coupling matrix through the permeability μ and permitivitty ε of the surrounding medium.

So far the development is quite standard and well known. However, what distinguishes the coupled wave model from the ordinary multiple conductor transmission line is the boundary conditions and their representation by an interconnection matrix $\overline{T}$. In a uniformly wound coil of M turns, regardless of shape, the physical input terminals of the coil as a unit are taken to be at $\zeta=-L$ on the first turn and at $\zeta=L$ on the M^{th} turn, V_t and I_t being the terminal voltage and current demanded by external connections. The periodic structure of the coil, on the other hand, demands equality of the voltage and current at $\zeta=-L$ on the m^{th} turn to those at $\zeta=L$ on the $(m-1)^{th}$ turn. In brief, these conditions are effectively expressed by the vector equations

$$\overline{T}\ \underline{I}(-L,s) - \underline{I}(L,s) = 0 \quad (5)$$

$$\overline{T}\ \underline{V}(-L,s) - \underline{V}(L,s) = V_t\underline{a}_M \quad (6)$$

where

$$\underline{a}_M = \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ 0 \\ 1 \end{bmatrix}$$

and $\overline{T}$ is the interconnection matrix. In a uniformly wound coil, with no folding layers, it takes the particular form

$$\overline{T} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & 0 & \dots & 0 \\ 0 & \cdot & \cdot & \cdot & \cdot & \dots & 1 \\ 1 & 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix} \quad (6)$$

More generally, every uniform coil, including those with folding layers, has an interconnection matrix whose form is such that every column and every row has but one nonzero element. Such matrices are orthogonal having eigenvalues that lie on the unit circle in the complex plane which are intimately related to the resonances normally observed in any coil.

The two-turn coil is the simplest symmetric configuration which permits straight forward exact solution of the boundary value problem consisting of Eqs. (1)-(6). Such solution is analyzed in detail in the earlier work[3]. The general case of M-turn coil, however, poses formidable difficulties owing to the necessity for tedious algebraic manipulation of MxM matrices. Nevertheless, some broad features can be deduced without explicit calculation.

A significant parameter, especially for our purpose here in light of the experimental data, is the input terminal admittance Y_t since it plays a dominant role in the way the coil is interfaced with external systems. As usual, it is defined as

$$Y_t = \frac{I_t}{V_t} = \frac{I_1(-L,s)}{V_t(s)}. \quad (7)$$

The remarkable departure of the coupled wave model from circuit theory appears in this admittance function. If the coil were viewed as a lumped inductance L_o, its admittance would be $1/sL_o$. Even a representation by a finite number of hypothetical capacitance-inductance meshes, as commonly employed in analysis of transformers, would still yield an admittance which is the ratio of rational polynomials in s. By constrast, the admittance predicted by the coupled wave model is a function not of s but of the delay parameter $e^{-s\tau}$, where $\tau = 2L/c$ is the travel time around a single turn. It can be shown that in general the admittance may be expanded in a series

$$Y_t = a_o + \sum_{n=1}^{\infty} a_n\ e^{-ns\tau} \quad (8)$$

where a_n are constant coefficients determined by the coupling matrix elements Y_{ij}. Significantly, the exponential terms indicate that voltages and currents, as observables, evolve not as continuous functions in time but in discrete steps having durations equal to the characteristic travel time around a single turn. Even in the case of three turns, however, calculation of the coefficients a_n for a given matrix Y is not a simple task. Nevertheless, the first four coefficients have been derived for the purpose of comparing to the experimental observations in broad terms. In this case, the coil is appropriately characterized by the characteristic wave admittance matrix which takes the general form

$$\overline{Y} = \begin{bmatrix} Y_1+Y_3 & -Y_2 & -Y_3 \\ -Y_2 & Y_1+Y_2 & -Y_2 \\ -Y_3 & -Y_2 & Y_1+Y_3 \end{bmatrix} \quad (9)$$

while the interconnection matrix reduces to

$$\overline{T} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad (10)$$

The 3-turn coil is useful for the purpose of preliminary experimental verification because it does not suffer from degeneracy as the 2-turn coil while at the same time it holds the calculations to reasonable proportions.

Theory of Experimental Method

Observation of the effects of the terminal admittance function offers an excellent route to verification and appraisal. Thus, the core of the experimental method consists of time domain reflectrometry (TDR) both in pulse and step form. It is well known that when a cable is terminated with an admittance $Y_t(s)$ as the load, the reflection coefficient in terms of normalized admittance is given by

$$\rho(s) = \frac{1-y_t}{1+y_t}, \quad y_t = Y_t/Y_o \quad (11)$$

where Y_o is the characteristic admittance of the cable, in this case 1/50 mho. Moreover, if $V_g(s)$ is the equivalent voltage (as amplitude of the electric field at a reference terminal plane) of a matched generator, then the reflected voltage observed at the generator terminal is given by

$$V_r(s) = \rho(s)V_g(s). \quad (13)$$

Obviously, in time domain $v_r(t)$ is a convolution of $v_g(t)$ with the inverse of $\rho(s)$. Alternately, the inversion can be effected by using the series form of the admittance function (8) to expand the reflection coefficient $\rho(s)$ in (11) into another series

$$\rho(s) = \rho_o[1 + \sum_{n=1}^{\infty} \rho_n e^{-ns\tau}]. \quad (14)$$

If $v_g(t)$ is arranged to be non-zero only over an interval shorter than τ, then a term-by-term inversion of (13) using (14) yields the sequence

$$v_r(t) = \rho_o v_g(t) + \rho_o \sum_{n=1}^{\infty} \rho_n v_g(t-n\tau). \qquad (15)$$

For the case $v_g(t)=u(t)$, the unit step function, the inversion becomes a sequence of weighted but successively delayed steps

$$v_r(t) = \rho_o u(t) + \rho_o \sum_{n=1}^{\infty} \rho_n \, u(t-n\tau). \qquad (16)$$

Equations (15) and (16) are the analytical expressions of the primary observables presented below, and they emphasize the significant departure from the circuit theoretic inductance model. In particular, if the circuit admittance $Y_t=1/sL_o$ were employed in (11) and (13), the reflected voltage for unit step input would be a single decaying exponential which is far different from the sequence (16) on short time scale. It is remarkable, however, to note that because ρ_n are monotonically decreasing, on a coarse time scale larger than τ, the sum in (16) approaches an exponential decay whose time constant is determined by the system resistance which in this case is principally the generator resistance rather than the much smaller coil resistance. This asymptotic behavior has been verified by computer simulation and is born out in the data below. For the sake of completeness, the first four coefficients which are calculated from the matrices (9) and (10) for the 3-turn coil are listed here in terms of normalized elements $y_i = Y_i/Y_o$:

$$\rho_o = (2-y_o)/(2+y_o) \qquad \rho_1 = 8y_2/p$$

$$\rho_2 = 4y_2^2 \, y_3/p \qquad \rho_3 = -8y_o/p$$

$$y_o = y_1 + y_3 \qquad p = 4 - y_o^2 .$$

Note that the second and third terms, ρ_1 and ρ_2, are positive and depend on the mutual turn-to-turn coupling while the fourth term is negative and principally dependent on self coupling corresponding to the diagonal terms of the Y matrix.

TDR Experiments

Time domain reflectrometry experiments were conducted on a 3-turn coil consisting of ordinary No. 22 enameled copper wire. The coil is in the shape of a race track (or paper clip) lying in a plane and measuring 100 cm in length and 15 cm in width, the ends being semicircles with 15 cm diameter. The overall peripheral length of each turn amounts to 220 cm which is the equivalent of approximately 7.3 nanoseconds travel time per turn. Choice of these dimensions is dictated by considerations of pulse resolution, given that the pulse width of the generator used is approximately 5 nanosecond.

A schematic diagram for the TDR measurements is shown in Figure 1. Briefly, a voltage pulse emerging from the generator is split in two, one half going to the coil along the main line, and the other half going to the oscilloscope along the auxiliarly line. The fixed delay suffered along these lines is common to all pulses and thus it is inconsequential. The main results are summarized next:

1. In Figures 2a and 2b, the leftmost faint trace is the incident pulse. The next pulse from the left is the first reflected pulse due to the discontinuity at the coil terminals, and it is proportional to the first term ρ_o in Eq. (15). If there were no turn-to-turn coupling, the incident pulse entering terminal 1 as positive and terminal 2 as negative travels the entire course of three turns amounting to a delay of approximately 22 nanosecond, arriving back at the terminals in inverted polarity, and thence back to the generator and oscilloscope. This appears as the inverted pulse in the trace, and it corresponds to the fourth term ρ_3 in Eq. (15). Because of coupling, however, immediately upon arrival at the outer turns, the incident pulse induces secondary ones having reversed polarity in the inner second and third turns. These secondary pulses travel along one turn and two turns, respectively, before arriving at the terminals on their way towards the generator in the same polarity as the incident pulse. The result is the two positive echo pulses appearing in the traces at approximately 7.5 and 15 nanosecond after the first reflection. These correspond to the terms ρ_1 and ρ_2 in Eq. (15) which are dependent on the off-diagonal elements of the coupling matrix. It is interesting to note that coupling beyond nearest neighbors is significant, because if γ_3 and therefore y_3 were zero the echo at 15 nanosecond would not be present.

2. The step response on short time scale appears in Figure 3a and 3b. The events are similar to the pulse case, save for the fact that the sequence is cumulatively additive. The magnitude of each of the stpes is proportional to the respective terms ρ_n in Eq. (16).

3. Figures 4a and 4b illustrate in a spectacular way the asymptotic behavior of the coil over long time scale, which approaches the exponential decay expected from circuit theory. In the wave viewpoint, however, the rate of decay is essentially a manifestation of the rate of decrease of the coefficients ρ_n as n increases

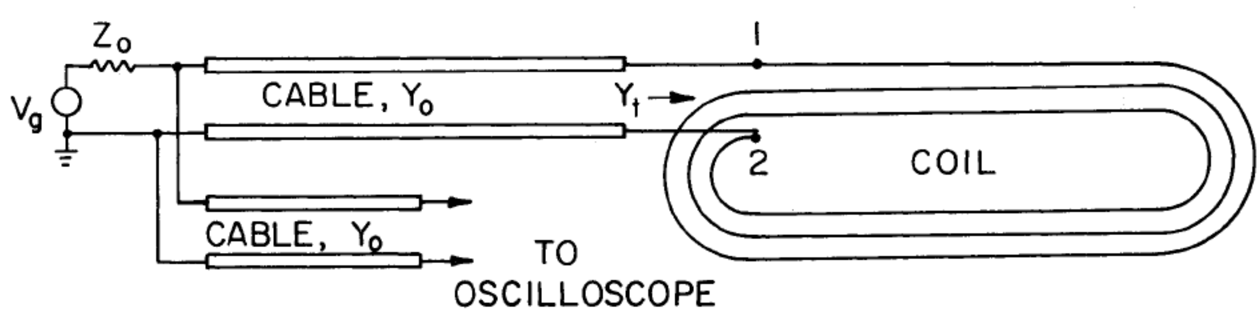
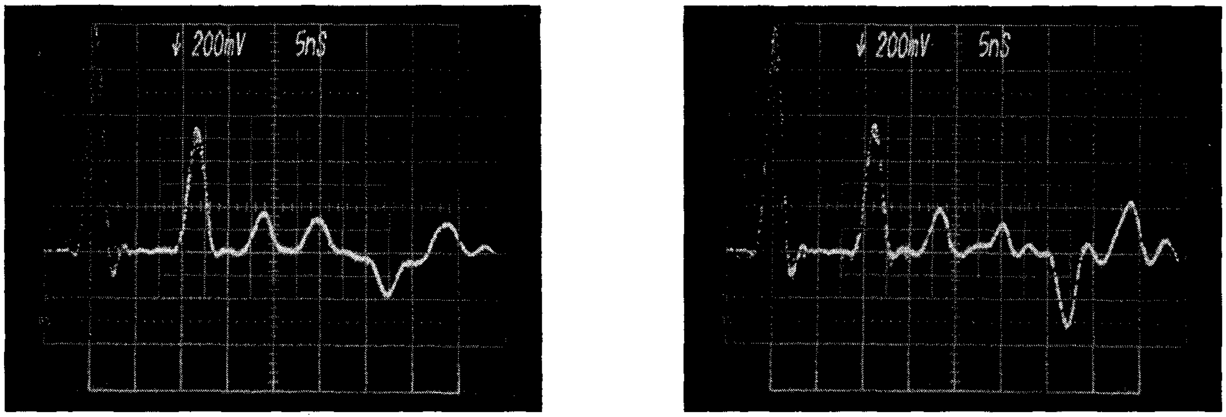
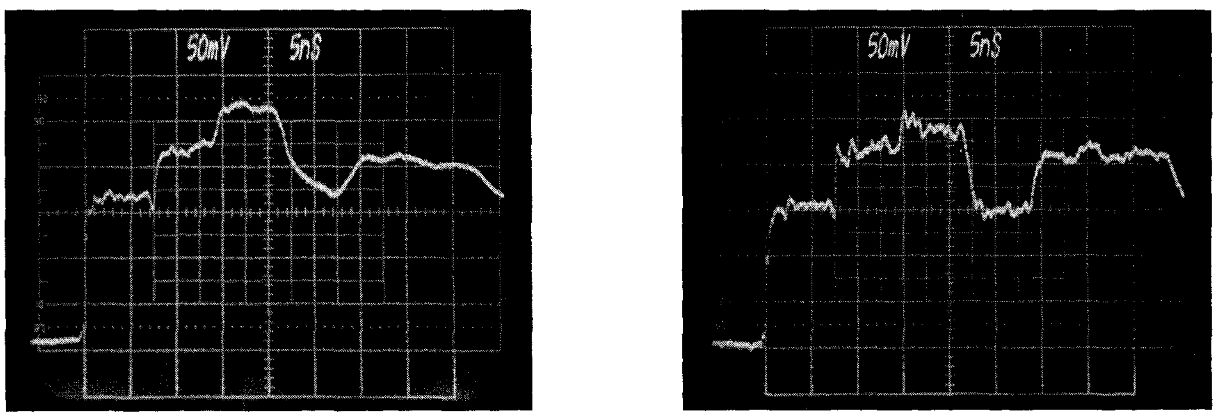


Figure 1: Schematic Diagram of TDR Arrangement and Coil



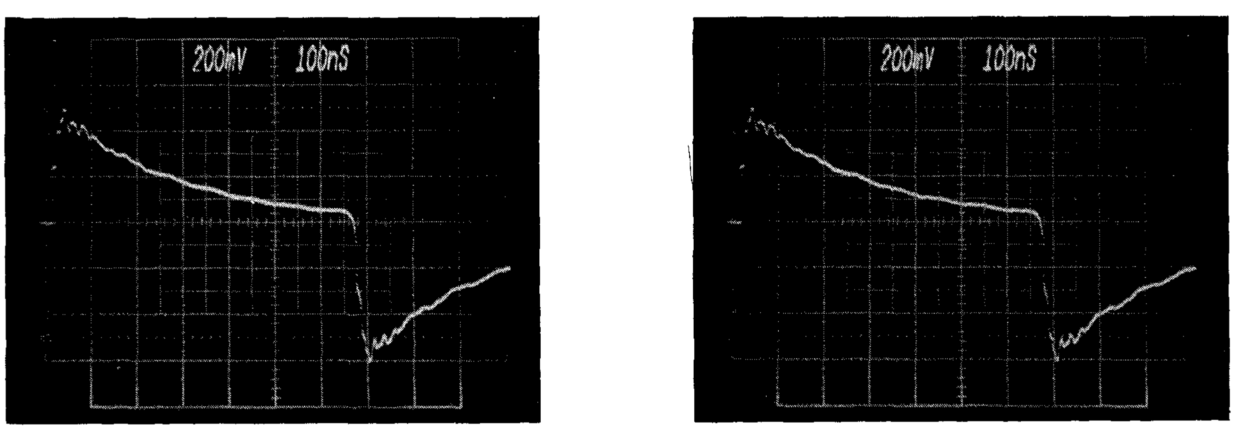
(a) (b)

Figure 2: Pulse reflection. a) Room temperature; b) Liquid nitrogen temperature



(a) (b)

Figure 3: Step reflection. a) Room temperature; b) Liquid nitrogen temperature



(a) (b)

Figure 4: Asymptotic long time reflection. a) Room temperature; b) Liquid nitrogen temperature

indefinitely. This rate is determined by the ratio of the admittance elements to the generator conductance (which is matched to the cable admittance Y_0). If the coil and generator are treated from a circuit standpoint, taking L_0 in series with the generator resistance $1/Y_0$, then clearly the decay time constant would indeed be L_0Y_0.

4. Operation at liquid Nitrogen temperature reduces the finite resistance of the coil wire. The resulting reduction in damping is mainly evident in the short time behavior of the coil as shown in Figure 2b for pulse input, and in Figure 3b for step input. The effect is most pronounced in the inverted echo which, as indicated above, travels the entire length of 3-turns. This echo is principally determined by the element y_0 of the admittance matrix. Theoretical considerations based on a minimum energy principle[4] indicate that the effect of finite wire resistance should appear as a first order additive correction on only the diagonal elements of the matrix which is consistent with the observed result. On the other hand, Figures 4a and 4b show no discernible difference in the long term decay rates at the two different temperatures. As indicated in 3 above, the generator resistance of 50 ohm, being much larger than the coil resistance both at room and at liquid Nitrogen temperatures, is the dominant resistance in determining the long term decay rate. Thus, this result is also consistent.

Conclusion

In summary, the travel time of a disturbance around a single turn and the turn-to-turn coupling are the dominant aspects by which the Coupled Wave Model departs from circuit theory in describing transient response of coils over short time scale. The excellent agreement of the model with the experimental observations enumerated above motivates confidence in the basic suppositions undelying its theory. As to the problem of high voltage breakdown, it is most likely to occur on the short time scale comparable to travel time per turn, as indicated by computer calculation on a two-turn coil[3]. Success of the model thus far makes it well worth the effort to tackle the more complicated and difficult configurations with a view toward developing refinements that would permit prediction and simulation.

Acknowledgement

Support of this work under the Scholarly Research Program, U.S. Air Force Office of Scientific Research, is gratefully acknowledged. It is a particular pleasure to thank Dr. Charles Oberly, Aero Propulsion Laboratory, Wright-Patterson AFB, for his continued interest and encouragement.

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